Int. J. Solids Structures, 1973, Vol. 9, pp. 693 to 695. Pergamon Press. Printed in Great Britain

COMMENTS ON THE PAPER: ADJOINT VARIATIONAL METHODS IN NONCONSERVATIVE STABILITY PROBLEMS [1]

THE authors have presented an interesting extension of the use of adjoint variational methods in nonconservative stability problems. The present comments do not concern the new work but rather the authors' comments on other approximate methods.

Prasad and Herrmann state that the local potential method has not been used to treat nonconservative stability problems. However, the present writer, in a 1966 paper [2], gave a formulation of Hamilton's Principle for nonconservative mechanical systems which is essentially a local potential method. The new formulation of Hamilton's Principle makes explicit use of the fact that during the virtual displacement of a mechanical system the forces acting on the system do not vary *even* if they are displacement dependent. Consequently, certain "displacement" terms in the functional of the extended Hamilton's Principle are not varied, just as certain temperature terms are not varied when the local potential method is applied to thermal systems. It would seem that the physical basis for the writer's work, the principle of virtual work, is as clear as the physical basis for the local potential method when used for thermal systems.

The present writer's work has been used as the basis of a finite element treatment of nonconservative problems of elastic stability by Barsoum [3].

REFERENCES

[1] SHYAM N. PRASAD and GEORGE HERRMANN, *Int. J. Solids Struct.* **8**, 29–40 (1972).
[2] MARK LEVINSON, *ZAMP.* **17**, 431–442 (1966).
[3] R. S. BARSOUM, *Int. J. Num. Meth. Eng.* **3**, 63–87 (1971).

Department of Civil Engineering and Engineering Mechanics MARK LEVINSON
McMaster University
Hamilton
Ontario
Canada

REPLY TO THE COMMENTS BY M. LEVINSON CONCERNING THE PAPER: ADJOINT VARIATIONAL METHODS IN NONCONSERVATIVE STABILITY PROBLEMS

THE authors would like to thank Dr. M. Levinson for his comments.

The concept of local potential includes in the Lagrangian, functions evaluated at the steady state and thus not subject to variation. This Lagrangian is a negative semidefinite

field, ψ, which is a function of the state variables defining the system and their time derivatives. With the aid of the governing differential equations of motion, the integral of ψ over the domain is then evaluated to first order in the functional neighborhood of the steady state. It is then possible to separate the time differentiation from the integration, obtaining

$$\int \psi \, dv = \frac{\partial \Phi}{\partial t} \leq 0$$

where Φ is termed a local potential and is in the nature of a generalized rate of entropy production. For Beck's problem, Φ may be defined as

$$\Phi(u, u^0) = \int_0^1 \left\{ \frac{\partial^2 u}{\partial x^2} \frac{\partial^2 u^0}{\partial x^2} + Fu \frac{\partial^2 u^0}{\partial x^2} + \gamma \frac{\partial u^0}{\partial t} u + m \frac{\partial^2 u^0}{\partial t^2} u \right\} dx$$

so that

$$\delta\Phi = \int_0^1 \left(\frac{\partial^4 u^0}{\partial x^4} + F \frac{\partial^2 u^0}{\partial x^2} + \gamma \frac{\partial u^0}{\partial t} + m \frac{\partial^2 u^0}{\partial t^2} \right) \delta u \, dx.$$

Therefore,

$$\delta\Phi = 0$$

when

$$\frac{\partial^4 u}{\partial x^4} + \beta \frac{\partial^2 u}{\partial x^2} + \gamma \frac{\partial u}{\partial t} + m \frac{\partial^2 u}{\partial t^2} = 0$$

with

$$u(0, t) = \frac{\partial u(0, t)}{\partial x} = \frac{\partial^2 u(1, t)}{\partial x^2} = \frac{\partial^3 u(1, t)}{\partial x^3} = 0$$

The above is obtained with an *a posteriori* subsidiary condition

$$u = u^0.$$

Further, if Φ is to be employed to form a basis of approximate solution, we may consider

$$u = \sum_n a_n(t) \varphi_n(x)$$

$$u^0 = \sum_n a_n^0(t) \varphi_n(x)$$

where $\varphi_n(x)$ are trial functions satisfying $\varphi(0) = \varphi'(0) = \varphi''(1) = \varphi'''(1) = 0$. This set of trial functions when substituted into the local potential Φ and minimized with respect to a_n yields with the subsidiary condition

$$\{a_n(t)\} = \{a_n^0(t)\}$$

a system of linear, ordinary, differential equations for $a_n(t)$. This system of equations is the same as that when the Galerkin method is used directly to obtain an approximate solution of the equation of motion of Beck's problem.

In the paper by M. Levinson (reference [2] of the comments), no such considerations as outlined here are found. On the other hand, the authors fail to see a physical basis of the method as proposed by the discusser.

Department of Civil Engineering,
University of Mississippi,
Mississippi 38677
U.S.A.

SHYAM N. PRASAD
GEORGE HERRMANN